

Shedding light on the pion production in heavy-ion collisions and application into the neutron star matter properties

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Within the framework of the quantum molecular dynamics transport model, the pion production and constraint of the high-density symmetry energy in heavy-ion collisions near threshold energy have been thoroughly investigated. The energy conservation in the decay of resonances and reabsorption of pions as well as in the inelastic nucleon-nucleon and nucleon-resonance collisions are taken into account. The isospin diffusion in the low-density region ($0.2\rho_0 - 0.8\rho_0$) and high-density region ($1.2\rho_0 - 1.8\rho_0$) is investigated by analyzing the kinetic energy spectra of neutron/proton and π^-/π^+ ratios in the isotopic reactions of $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn} + ^{112}\text{Sn}$ at the incident energy of 270 MeV/nucleon. The controversial conclusion of the π^-/π^+ ratio for constraining the high-density symmetry energy by different transport models with the FOPI data has been clarified by the complementary isospin observables in the dense medium and at the freeze-out stage. A soft symmetry energy with the slope parameter of $L(\rho_0) = 42 \pm 25$ MeV by using the standard error analysis within the range of 1σ is obtained by analyzing the experimental data from the S π RIT collaboration. The neutron stars with the maximal mass of $2 M_\odot$ and radius of 11-13 km are obtained with the constrained symmetry energy from the double ratio spectra of π^-/π^+ in the isotropic reactions.

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I. INTRODUCTION

Heavy-ion collisions at intermediate energies under the extreme conditions, such as the high density, high temperature, and high isospin asymmetry, provide a good environment for exploring the properties of dense nuclear matter. The particle production in heavy-ion collisions brings the information of the hot and compressed nuclear matter. Pion meson has been attracted attention for probing the dense matter, which was predicted by Yukawa as a propagator of the strong interaction [1] and later discovered by observing the cosmic rays in experiments [2]. Nowadays, pions might be created in laboratories via different reactions, such as heavy-ion collisions, photon, lepton and hadron induced reactions, electron-positron colliding etc. Particles produced in the GeV energy range manifest the information of the high-density nuclear matter and might be modified by the nuclear medium [3]. The nuclear equation of state (EOS) is expressed with the energy per nucleon as $E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + \mathcal{O}(\delta^4)$ in terms of baryon density $\rho = \rho_n + \rho_p$ and relative neutron excess $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, where $E_{\text{sym}}(\rho)$ is the symmetry energy [4–6]. The symmetry energy at the subsaturation density has important application in understanding the structure of weakly bound nuclei, nucleon-nucleon correlation, pasta structure of neutron star etc, which was extensively investigated by different approaches, such as the Pygmy dipole resonance, heavy-ion collision, fast fission, electron-nucleus scattering etc [7–16]. The high-density symmetry energy is related to the issues of com-

pact stars, such as the phase transition, binary neutron star merging, tidal deformation etc [17–22], which is still not well understood up to now. To extracting the density dependence of symmetry energy, new experiments are being carried out at the facility in the world, such as Radioactive Isotope Beam Facility (RIBF) in Japan [23], Rare Isotope Science Project in Korea (RAON) [24], Facility for Rare Isotope Beams (FRIB) in the USA [25], the Cooling Storage Ring (CSR) and the High Intensity Accelerator Facility (HIAF) in China [26].

It has been performed a number of experimental data for the pion production in heavy-ion collisions, such as the BEVALAC with the Berkeley Streamer Chamber [27, 28], the DIOGENE at the Saturne synchrotron in Saclay [29], the FOPI collaboration at the beam energies from 0.4A to 1.5A GeV at Gesellschaft für Schwerionenforschung [30], HIRFL-CSR in Lanzhou [31], the S π RIT collaboration at RIKEN for the reactions of $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn} + ^{112}\text{Sn}$ at 270A MeV [32]. It is well known that the opposite conclusion for constraining the high-density symmetry energy was obtained by analyzing the FOPI data of the π^-/π^+ excitation function with the different transport models [33, 34], namely the ‘pion puzzle’. At the near threshold energy, the pion mesons are mainly produced via the decay of $\Delta(1232)$ resonance. The pion transportation in heavy-ion collisions is modified in the nuclear medium, i.e., the pion-nucleon potential, in-medium cross section, threshold energy, decay width of resonance, reabsorption process via the reaction $\pi N \leftrightarrow \Delta$ etc. The influence of the π potential on the pion dynamics in heavy-ion collisions has been investigated

via transport models [35–42]. Recently, the uncertainties of observables in transport models for heavy-ion collisions have been discussed by the transport model evaluation project (TMEP) [43, 44]. More sophisticated investigation on the pion production in heavy-ion collisions is still expected, i.e., the isospin, density and momentum dependent pion-nucleon potential, resonance-nucleon interaction, in-medium properties of resonance, threshold energy correction etc. A unified description of symmetry energy effect on the neutron/proton and π^-/π^+ spectra is helpful for deeply understanding the experimental data.

In this work, the pion production in the isotopic nuclear reactions near threshold energy and high-density symmetry energy are to be investigated with the Lanzhou quantum molecular dynamics (LQMD) transport model. The article is organized as follows. In Sec. II, we briefly introduce the theoretical approach. The calculated results are shown in Sec. III. A summary perspective on the future experiments is given in Sec. IV.

II. THEORETICAL APPROACH

In the LQMD transport model, the production of resonances, hyperons and mesons is coupled to the meson-baryon and baryon-baryon collisions, in which the nuclear dynamics in heavy-ion collisions and hadron induced reactions have been extensively investigated [14, 45–47]. The temporal evolutions of nucleons and nucleonic resonances are described by Hamilton’s equations of motion under the self-consistently generated two-body and three-body potentials with the Skyrme effective interaction. The symmetry energy is composed of three parts, namely, the kinetic energy from Fermi motion, the local density-dependent interaction, and the momentum-dependent potential, which reads as

$$E_{sym}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left(\frac{3}{2} \pi^2 \rho \right)^{2/3} + E_{sym}^{loc}(\rho) + E_{sym}^{mom}(\rho). \quad (1)$$

The stiffness of symmetry energy is adjusted by

$$E_{sym}^{loc}(\rho) = \frac{1}{2} C_{sym} (\rho/\rho_0)^{\gamma_s}. \quad (2)$$

The parameter C_{sym} is 52.5 MeV, and the stiffness parameter γ_s is adjusted for getting the density dependence of symmetry energy, e.g., the values of 0.3, 1, and 2 being the soft, linear, and hard symmetry energy, corresponding to the slope parameters $[L(\rho_0) = 3\rho_0 dE_{sym}(\rho)/d\rho|_{\rho=\rho_0}]$ of 42, 82, and 139 MeV, respectively. As shown in Fig. 1(a), it can be seen that in all cases of symmetry energy at the saturation density ($\rho_0 = 0.16 \text{ fm}^{-3}$) with the value of 31.5 MeV. The different slope of symmetry energy influences the neutron/proton ratio and also the mass-radius relation of neutron star. The hard symmetry energy ($L=139 \text{ MeV}$)

results in the more repulsive force for neutrons in the neutron-rich matter and consequently leads to the lower neutron/proton ratio in the high-density domain and to the larger free neutron/proton ratio. The symmetry energy dominates the isospin diffusion in heavy-ion collisions and leads to the isospin density difference. The larger neutron/proton ratio in the nuclear matter with the density regime of $1.2 \leq \rho/\rho_0 \leq 1.8$ with the soft symmetry energy in the temporal evolution of $^{132}\text{Sn}+^{124}\text{Sn}$ is obtained as shown in Fig. 1(b), in particular in the regime of kinetic energy above 100 MeV. The supra-saturation densities $1.2 \leq \rho/\rho_0 \leq 1.8$ of protons and neutrons are accumulated from the averaged temporal evolution and judged from the local density formed in heavy-ion collisions. The stiffness of symmetry energy might be constrained by the isospin observables produced different density region in heavy-ion collisions.

The reaction channels for the pion production contributed from the resonance decay and the direct process, such as $\Delta(1232)$, $N^*(1440)$, $N^*(1535)$ etc, are included in the model as follows [14]

$$\begin{aligned} NN &\leftrightarrow N\Delta, \quad NN \leftrightarrow NN^*, \quad NN \leftrightarrow \Delta\Delta, \\ \Delta &\leftrightarrow N\pi, N^* \leftrightarrow N\pi, NN \leftrightarrow NN\pi(\text{s-state}), \end{aligned} \quad (3)$$

Here the nucleon-nucleon (resonance) and pion-nucleon collisions are treated as the stochastic and isotropic scattering in the temporal evolution of reaction system. The momentum-dependent decay widths are implemented into the model for the resonances of $\Delta(1232)$ and $N^*(1440)$ and the elementary cross sections are taken from the parameterized formula calculated by the one-boson exchange model [48]. The constant width of $\Gamma=150 \text{ MeV}$ for the $N^*(1535)$ decay is used in the calculation. The elastic scatterings in nucleon-nucleon, nucleon-resonance ($NR \rightarrow NR$) and resonance-resonance ($RR \rightarrow RR$) collisions and inelastic collisions of nucleon-resonance ($NR \rightarrow NN$, $NR \rightarrow NR'$) and resonance-resonance channels ($RR \rightarrow NN$, $RR \rightarrow NR$, $RR \rightarrow RR'$, R and R' being different resonances), have been included in the model. The direct process $NN \leftrightarrow NN\pi(\text{s-state})$ roughly contributes 15% pion yields and the cross sections are taken as the same with the Giessen Boltzmann-Uehling-Uhlenbeck (GiBUU) transport model [49].

The transportation of pion in nuclear medium is also described by the Hamiltonian equation of motion as

$$H_M = \sum_{i=1}^{N_M} [V_i^{Coul} + \omega(\mathbf{p}_i, \rho_i)]. \quad (4)$$

The Coulomb interaction is given by

$$V_i^{Coul} = \sum_{j=1}^{N_B} \frac{e_i e_j}{r_{ij}} \quad (5)$$

with $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. Here the N_M and N_B are the total numbers of mesons and baryons including charged resonances, respectively. It should be noticed that the pion

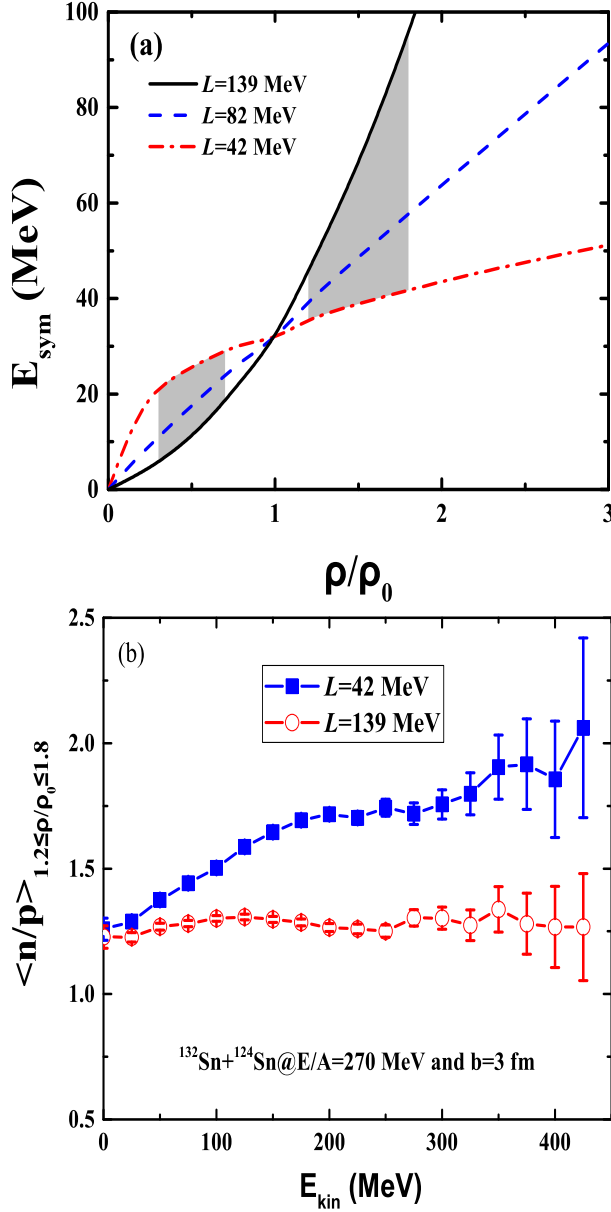


FIG. 1. (a) The density dependence of nuclear symmetry energy with different stiffness and (b) the neutron/proton ratio at the supra-saturation densities $1.2 \leq \rho/\rho_0 \leq 1.8$ with the temporal evolutions in collisions of $^{132}\text{Sn}+^{124}\text{Sn}$ at the beam energy of 270 MeV/nucleon and with the impact parameter of $b=3$ fm.

meson is taken as the point particle and the Coulomb interaction between mesons is neglected owing to the limited numbers in comparison with the baryons.

The pion energy in the nuclear medium is composed of the isoscalar and isovector contributions as

$$\omega_\pi(\mathbf{p}_i, \rho_i) = \omega_{\text{isoscalar}}(\mathbf{p}_i, \rho_i) + C_\pi \tau_z \delta (\rho/\rho_0)^{\gamma_\pi}. \quad (6)$$

Here the isovector coefficient $C_\pi = \rho_0 \hbar^3 / (4f_\pi^2) = 36$ MeV, isospin asymmetry $\delta = (\rho_n - \rho_p) / (\rho_n + \rho_p)$ and isospin splitting parameter $\gamma_\pi = 2$. The isospin quantities are set

to be $\tau_z = -1, 0$, and 1 for π^+ , π^0 , and π^- , respectively [39]. The isoscalar part $\omega_{\text{isoscalar}}$ is estimated by the Δ -hole model [50, 51].

The energy balance in the decay of resonances and re-absorption of pions in nuclear medium is satisfied by the relation $R \leftrightarrow N\pi$ (R being the resonance) as

$$\begin{aligned} \sqrt{m_R^2 + \mathbf{p}_R^2} + U_R(\rho, \delta, \mathbf{p}_R) &= \sqrt{m_N^2 + (\mathbf{p}_R - \mathbf{p}_\pi)^2} \\ &+ U_N(\rho, \delta, \mathbf{p}) + \omega_\pi(\mathbf{p}_\pi, \rho) + V_{\pi N}^{\text{Coul}}. \end{aligned} \quad (7)$$

The \mathbf{p}_R and \mathbf{p}_π are the momenta of resonance and pion, respectively. The term $V_{\pi N}^{\text{Coul}}$ has the contribution only for the charged pair channels of $\Delta^0 \leftrightarrow \pi^- + p$ and $\Delta^{++} \leftrightarrow \pi^+ + p$, and no effect for the channels associated with the π^0 and neutron production. The optical potential can be evaluated from the in-medium energy $V_\pi^{\text{opt}}(\mathbf{p}, \rho) = \omega_\pi(\mathbf{p}, \rho) - (m_\pi^2 + \mathbf{p}^2)^{1/2}$. The U_R and U_N are the single-particle potentials for resonance and nucleon, respectively. The vacuum spectral function for the resonance production and decay is used in this work. Similarly, the energy conservation is also treated for the resonance production and the direct s-state creation, e.g., the collision with the local baryon density ρ and the isospin asymmetry δ as

$$\begin{aligned} \sqrt{m_N^2 + \mathbf{p}_1^2} + U_N(\rho, \delta, \mathbf{p}_1) + \sqrt{m_N^2 + \mathbf{p}_2^2} + U_N(\rho, \delta, \mathbf{p}_2) &= \\ \sqrt{m_R^2 + \mathbf{p}_R^2} + U_R(\rho, \delta, \mathbf{p}_R) + \sqrt{m_N^2 + \mathbf{p}'^2} + U_N(\rho, \delta, \mathbf{p}') &(8) \end{aligned}$$

for the channel $NN \leftrightarrow NR$ with the momentum conservation relation of $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_R + \mathbf{p}'$. For example, the $\Delta(1232)$ optical potential is calculated via the nucleon optical potential by [52]

$$\begin{aligned} U_{\Delta^-} &= U_n, \quad U_{\Delta^{++}} = U_p, \quad U_{\Delta^+} = \frac{1}{3}U_n + \frac{2}{3}U_p, \\ U_{\Delta^0} &= \frac{1}{3}U_p + \frac{2}{3}U_n, \end{aligned} \quad (9)$$

where the U_n and U_p are the single-particle potentials for neutron and proton, respectively. The N^* -nucleon potential is taken as the same with the nucleon-nucleon potential. The density, isospin and momentum dependent single-nucleon potential is obtained as follows

$$\begin{aligned} U_\tau(\rho, \delta, \mathbf{p}) &= \alpha \left(\frac{\rho}{\rho_0} \right) + \beta \left(\frac{\rho}{\rho_0} \right)^\gamma + E_{\text{sym}}^{\text{loc}}(\rho) \delta^2 \\ &+ \frac{\partial E_{\text{sym}}^{\text{loc}}(\rho)}{\partial \rho} \rho \delta^2 + E_{\text{sym}}^{\text{loc}}(\rho) \rho \frac{\partial \delta^2}{\partial \rho_\tau} \\ &+ \frac{1}{\rho_0} C_{\tau, \tau} \int d\mathbf{p}' f_\tau(\mathbf{r}, \mathbf{p}) [\ln(\epsilon(\mathbf{p} - \mathbf{p}')^2 + 1)]^2 \\ &+ \frac{1}{\rho_0} C_{\tau, \tau'} \int d\mathbf{p}' f_{\tau'}(\mathbf{r}, \mathbf{p}) \\ &\times [\ln(\epsilon(\mathbf{p} - \mathbf{p}')^2 + 1)]^2. \end{aligned} \quad (10)$$

Here $\tau \neq \tau'$, $\partial \delta^2 / \partial \rho_n = 4\delta \rho_p / \rho^2$ and $\partial \delta^2 / \partial \rho_p = -4\delta \rho_n / \rho^2$. The nucleon effective (Landau) mass in nuclear matter of isospin asymmetry $\delta = (\rho_n - \rho_p) / (\rho_n + \rho_p)$

with ρ_n and ρ_p being the neutron and proton density, respectively, is calculated through the potential as $m_\tau^* = m_\tau / \left(1 + \frac{m_\tau}{|\mathbf{p}|} \left| \frac{dU_\tau}{d\mathbf{p}} \right| \right)$ with the free mass m_τ at Fermi momentum $\mathbf{p} = \mathbf{p}_F$. The parameters α , β , γ and ρ_0 are set to be the values of -215.7 MeV, 142.4 MeV, 1.322 and 0.16 fm^{-3} , respectively. The $C_{\tau,\tau} = C_{mom}(1+x)$, $C_{\tau,\tau'} = C_{mom}(1-x)$ ($\tau \neq \tau'$) and the isospin symbols $\tau(\tau')$ represent proton or neutron. The values of 1.76 MeV, $5 \times 10^{-4} \text{ c}^2/\text{MeV}^2$ are taken for the C_{mom} and ϵ , respectively, which result in the effective mass $m^*/m=0.75$ in nuclear medium at saturation density for symmetric nuclear matter. The parameter x as the strength of the isospin splitting with the value of -0.65 is taken in this work, which has the mass splitting of $m_n^* > m_p^*$ in nuclear medium. A incompressibility modulus of $K=230$ MeV for isospin symmetric nuclear matter is obtained at the saturation density. Recently, the influence of the pion potential on pion dynamics in heavy-ion collisions has been extensively investigated with different transport models [38–41, 53–55].

III. RESULTS AND DISCUSSION

The isospin diffusion in heavy-ion collisions is associated with the gradient of isospin density ($\rho_n - \rho_p$) and of symmetry potential $U_{sym} = (U_n(\rho, \delta, \mathbf{p}) - U_p(\rho, \delta, \mathbf{p}))/2\delta$, in which the symmetry energy plays a significant role on the rearrangement of nucleons in the nuclear evolution. The neutron and proton density distributions vary with the evolution of reaction system and neutron/proton ratio is diverse in the different density range. To extract the high-density symmetry energy, the observables emitted from the high-density domain in nuclear collisions are expected. The neutron/proton ratio is a direct probe for extracting the high-density symmetry energy. But detecting the neutrons with high statistics in experiments is still a difficulty task. We calculated the kinetic energy spectra of neutron/proton (N/Z) ratio of free nucleons produced in collisions of $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn} + ^{112}\text{Sn}$ at 270A MeV with the different stiffness of symmetry energy as shown in Fig. 2. The free nucleons are judged via the minimum spanning tree (MST) procedure with the relative distance $r_0 = 3 \text{ fm}$ and relative momentum $p_0 = 200 \text{ MeV}/c$ in phase space [45]. It is obvious that the hard symmetry energy with $L=139 \text{ MeV}$ leads to the larger N/Z ratio, in particular at the energies above 100 MeV for the neutron-rich system. The results are consistent with the conclusion in Fig. 1 (b), which is caused from the more repulsive interaction for neutrons in the dense neutron-rich matter for the hard symmetry energy ($L=139 \text{ MeV}$). More free neutrons are created with the hard symmetry energy, which is opposite with the Fermi-energy heavy-ion collisions [13]. The conclusion is consistent with the IBUU04 calculation, in which the s-

tiffer symmetry energy in the high-density region leads to the enhancement of free neutron/proton ratio [56]. Hunting the measurable quantities for constraining the high-density symmetry energy is very necessary, e.g., π^-/π^+ , K^0/K^+ , Σ^-/Σ^+ etc.

The symmetry energy as a major ingredient in the nuclear equation of state plays a significant role on the mass-radius relation and maximal mass of neutron stars, phase transition from quark-gluon plasma to hadron phase, nuclear structure of rare isotopes etc. The high-density behavior of symmetry energy from the pion production in heavy-ion collisions has been extensively investigated via transport models, such as the isospin dependent Boltzmann-Uehling-Uhlenbeck (IBUU) and relativistic Vlasov-Uehling-Uhlenbeck (RVUU) [54, 57–59]. Recently, the transverse momentum spectra of pions produced in the isotopic reactions have been measured by the S π RIT collaboration at RIKEN [32, 60]. We systematically analyzed the pion production within the framework of LQMD transport model, in which the energy conservation in the decay and reabsorption of resonances has been implemented into the model. The density profile of primordial pions in collisions of $^{132}\text{Sn} + ^{124}\text{Sn}$ at 270A MeV is calculated as shown in Fig. 3. The black-solid line denotes the total pion production ($\pi^- + \pi^0 + \pi^+$). And the red-dashed and the blue-dotted-dashed lines represent π^- and π^+ , respectively. It is obvious that a number of pions are produced in the low-density region, but pions at the suprasaturation densities are still appreciable. The low-density pions is mainly caused from the rescattering processes, i.e., $\pi N \leftrightarrow \Delta(1232)$ and $\Delta(1232)N \leftrightarrow NN$ [55]. Therefore, to extract the high-density information of symmetry energy, one needs to distinguish the final pions from the different density range. It is known that the π^-/π^+ ratio of primordial pions without the rescattering process is approximately satisfied to the quadratic relation by the statistical model $\pi^-/\pi^+ = \frac{5n^2+np}{5p^2+np} \approx (n/p)^2$ [50]. To clarify the density range of primordial pions produced in nuclear collisions, shown in Fig. 4 is the kinetic energy spectra of π^-/π^+ ratio selected in the density region of $0.2 \leq \rho/\rho_0 \leq 0.8$ and $1.2 \leq \rho/\rho_0 \leq 1.8$, respectively. It is pronounced that the symmetry energy effect is opposite in the different density domain, namely, the larger π^-/π^+ ratio with the hard symmetry energy in the low-density region, however the lower π^-/π^+ value at high densities. Moreover, the average π^-/π^+ ratio in the low-density region of $0.2 \leq \rho/\rho_0 \leq 0.8$ is close to the square of N/Z ratio (2.43) of reaction system. But the π^-/π^+ ratio in the high-density region is approximately the average N/Z value (1.56) at the kinetic energies below 100 MeV. Therefore, the π^-/π^+ ratio from the total pion multiplicities is indistinguishable for extracting the high-density symmetry energy. The density profile of pions is influenced by the rescattering cross sections and pion transportation in nuclear medium, which might be different in

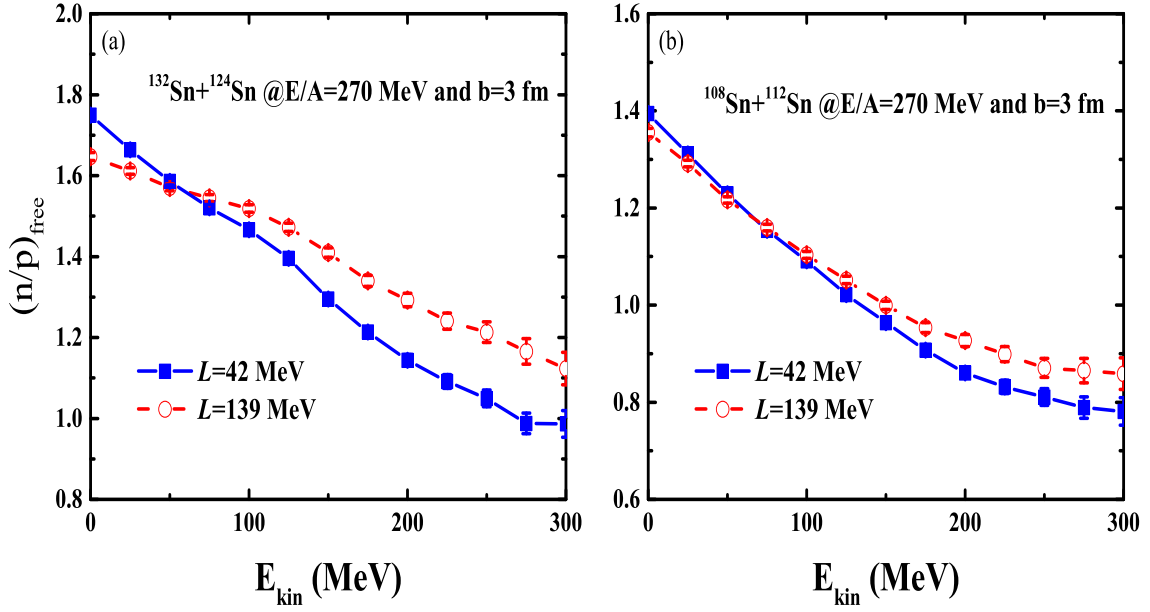


FIG. 2. The kinetic energy spectra of free neutron/proton ratio produced in collisions of $^{132}\text{Sn}+^{124}\text{Sn}$ (left panel) and $^{108}\text{Sn}+^{112}\text{Sn}$ (right panel), respectively.

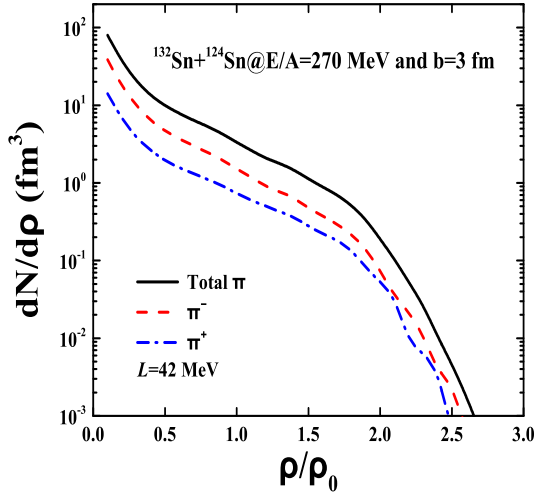


FIG. 3. The density profile of pion production in collisions of $^{132}\text{Sn}+^{124}\text{Sn}$ at 270A MeV.

the treatment in transport models although the similar total pion multiplicity. The kinetic energy or transverse momentum spectra of π^-/π^+ ratio with the cutting windows of rapidity and azimuthal emission might be available probes for constraining the high-density symmetry energy.

The phase-space distribution of emitted particles in heavy-ion collisions is influenced by the in-medium properties, i.e., the rapidity, transverse momentum, kinetic energy, invariant mass spectra etc. Shown in Fig. 5 is a comparison of the pion potential and threshold correction in collisions of $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn}+^{112}\text{Sn}$ at

the incident energy of 270A MeV. The energy conservation in the production of pion and resonance is related to the mean-field potentials of nucleons, pions and resonances. It is obvious that the inclusion of the energy conservation with Eqs (7) and (8) enhances the high-momentum pion production, which is caused from the fact that the energetic pions are created for satisfying the energy conservation in the resonance decay in Eq. (7) for the case with threshold correction and without π potential (blue-dashed line). The term $V_{\pi N}^{Coul}$ in Eq. (7) has the negligible contribution on the pion transverse momentum spectra by the Coulomb interaction between π^- and proton or π^+ and proton from the resonance decay. The pion-nucleon potential leads to the reduction of pion yields because of the attractive interaction, in particular for the π^+ production. The experimental data of transverse momentum spectra from the S π RIT collaboration [32] are nicely reproduced with the inclusion of energy conservation relation and pion potential. The symmetry energy effect is also analyzed as shown in Fig. 6. The effect is pronounced for the π^- production and the soft symmetry energy enhances the yields. The π^+ spectra weakly depend on the stiffness of symmetry energy because the proton-proton and proton-neutron collisions are almost not influenced by the symmetry energy in the neutron-rich system. It is concluded that π^-/π^+ ratio is also enhanced with the symmetry energy of $L=42$ MeV and consistent with the high-density results in Fig. 4(b). The high-transverse momentum pions are mainly produced from the high-density zone in nuclear collisions and might be probes of high-density symmetry energy, i.e. the pion produced above 150 MeV/c. It is noticed

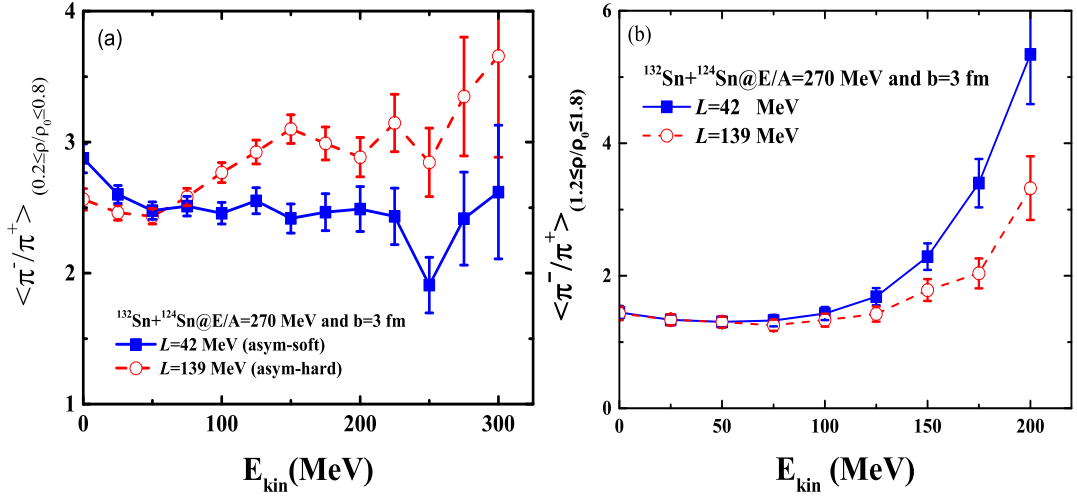


FIG. 4. The kinetic energy spectra of π^-/π^+ ratio in collisions of $^{132}\text{Sn}+^{124}\text{Sn}$ at 270 MeV within the baryon density region of (a) $0.2 \leq \rho/\rho_0 \leq 0.8$ and (b) $1.2 \leq \rho/\rho_0 \leq 1.8$, respectively.

that the π^- production is slightly underestimated by the LQMD model, which is caused from the in-medium neutron-neutron inelastic cross section and neutron-skin thickness in the initialization process for the neutron-rich nuclei.

The single ratio of π^-/π^+ in the reactions of $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn} + ^{112}\text{Sn}$ at 270 MeV is calculated as shown in Fig. 7. It is obvious that the soft symmetry energy with $L=42$ MeV is more consistent with the S π RIT data [32], in particular for the neutron-rich system $^{132}\text{Sn} + ^{124}\text{Sn}$. The results are self-consistent with the kinetic energy spectra of free neutron/proton ratio in Fig. 2, in which the hard symmetry energy leads to the larger neutron/proton ratio. The symmetry energy effect is more pronounced in the neutron-rich system in comparison with the neutron-poor system $^{108}\text{Sn} + ^{112}\text{Sn}$. To eliminate the Coulomb interaction and reduce some uncertainties, we analyzed the double ratios (DRs) in the isotopic reactions, which are defined as the relation $\text{DR}(n/p) = (n/p)_{^{132}\text{Sn}+^{124}\text{Sn}}^{\text{free}} / (n/p)_{^{108}\text{Sn}+^{112}\text{Sn}}^{\text{free}}$ and $\text{DR}(\pi^-/\pi^+) = (\pi^-/\pi^+)_{^{132}\text{Sn}+^{124}\text{Sn}} / (\pi^-/\pi^+)_{^{108}\text{Sn}+^{112}\text{Sn}}$, respectively. The method has been used for constraining the symmetry energy at the subsaturation density and the isospin splitting of nucleon effective mass in nuclear matter via heavy-ion collisions [61]. The symmetry energy effect is pronounced at the high kinetic energies (high transverse momenta) in the isotopic reactions as shown in Fig. 8, i.e., in the domain of $E_{\text{kin}} > 100$ MeV or $p_T > 150$ MeV/c. The double ratio of neutron/proton has more obvious symmetry energy effect in comparison with the case of π^-/π^+ , but the larger DR value for the pion production. The complementary observables manifest that the DRs of high kinetic energy or transverse momentum might be probes for extracting the high-density symmetry energy, i.e., the $\text{DR}(n/p)$ and $\text{DR}(\pi^-/\pi^+)$ spectra. The slope parameter of symmetry

energy at saturation density $L(\rho_0) = 42 \pm 25$ is obtained by using the standard error analysis method within the range 1σ . Due to the mixing of pions produced at high and low densities, the symmetry energy effect on the DR spectra is reduced. However, at high transverse momentum ($p_T \geq 150$ MeV/c) the DRs exhibit significant sensitivity to the high-density symmetry energy, and the difference is about 20%. It is noted that the constraint of $42 \text{ MeV} < L < 117 \text{ MeV}$ is obtained by the dcQMD calculations. Different with the dcQMD model, we fixed the isospin splitting of nucleon effective mass, i.e., $(m_n^* - m_p^*)/m_n = 0.04$ with the isospin asymmetry of $\delta = 0.2$ at the saturation density. The calculations from the dcQMD model manifest that the hard symmetry energy with $L(\rho_0) = 151$ MeV leads to the larger single ratio π^-/π^+ and DRs at the high transverse momenta, which is possibly caused from the larger pion-nucleon scattering cross section and the dominating contribution of the low-density pions on the transverse momentum spectra. In comparison with our previous results for constraining the high-density symmetry energy from the pion production in heavy-ion collisions [34], the isospin, density and momentum dependent nucleon-nucleon and pion-nucleon potentials, the energy conservation of resonance production and reabsorption with the channel of $NN \leftrightarrow NR$, and the energy conservation of pion production and reabsorption with the channel of $R \leftrightarrow \pi N$ have been implemented into the LQMD model. The modifications are essential for the transverse momentum spectra of pions near threshold energies. It is obvious that the symmetry energy effects of $\text{DR}(n/p)$ and $\text{DR}(\pi^-/\pi^+)$ spectra are opposite in the high kinetic energy or transverse momentum region. The larger values of $\text{DR}(n/p)$ and lower values of $\text{DR}(\pi^-/\pi^+)$ with the hard symmetry energy are complementary and manifest that the pions

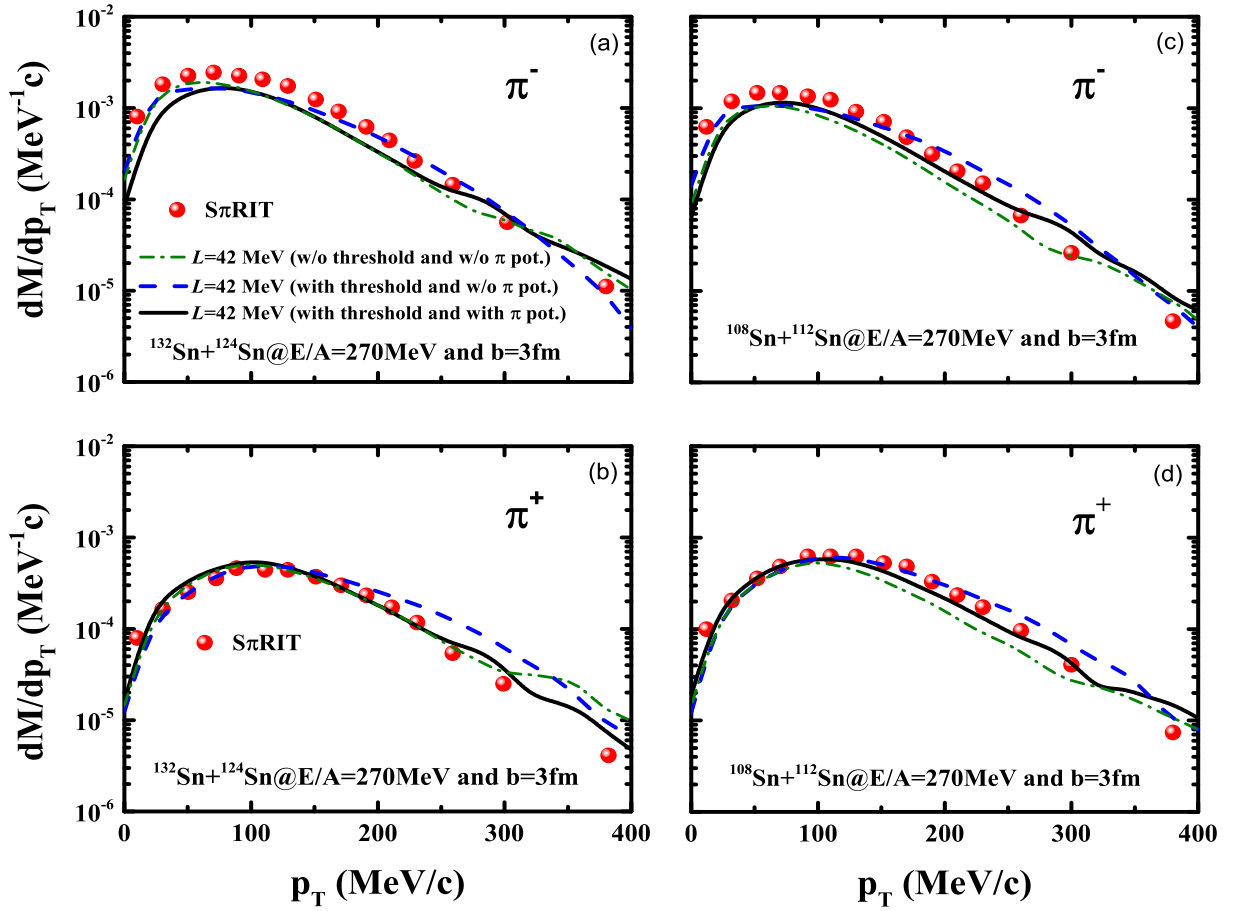


FIG. 5. Comparison of the transverse momentum spectra with the inclusion of threshold energy correction and pion potential in collisions of $^{132}\text{Sn}+^{124}\text{Sn}$ (left panel) and $^{108}\text{Sn}+^{112}\text{Sn}$ (right panel), respectively. The experimental data are taken from the S π RIT collaboration [32].

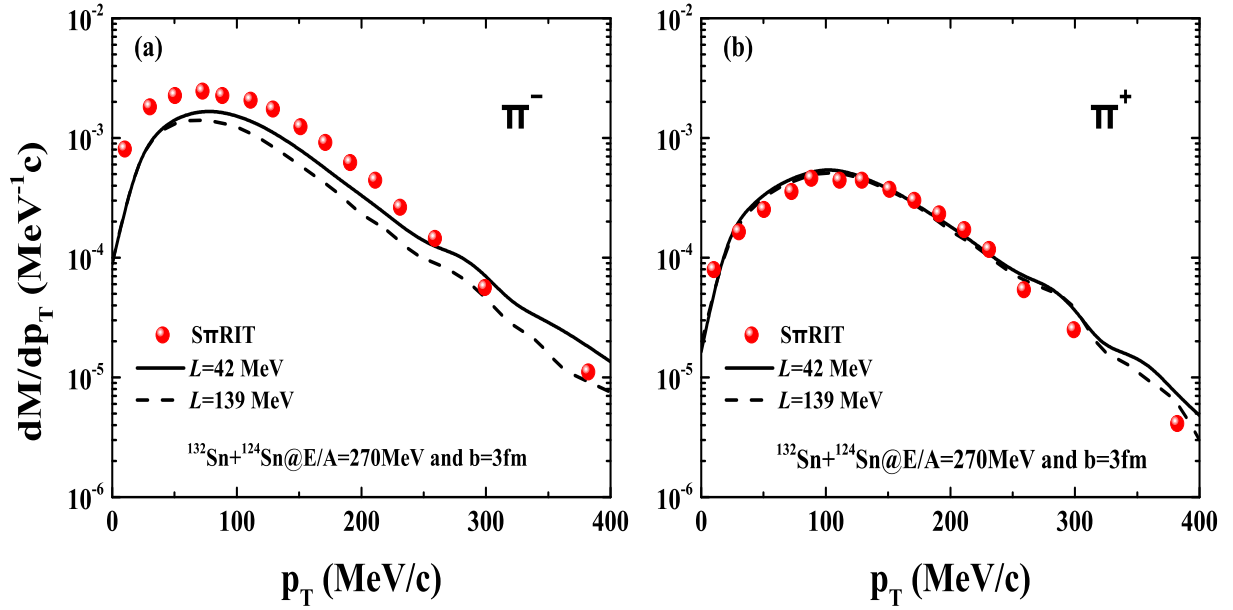


FIG. 6. The transverse momentum distribution of pion production with different symmetry energy in the reaction of $^{132}\text{Sn}+^{124}\text{Sn}$ at the beam energy of 270 MeV/nucleon and the impact parameter of 3 fm and compared with the available data from S π RIT collaboration [32].

in the high transverse momentum ($p_T \geq 150$ MeV/c) are mainly produced in the high-density domain. The results are also self-consistent with the kinetic energy spectra of the single ratios of $(n/p)_{free}$ and π^-/π^+ in the high-density domain.

The mass-radius relation of neutron stars (NSs) is influenced by the density dependent symmetry energy and properties of isospin symmetric nuclear matter. It has been investigated that the maximal mass of NSs is related to the incompressibility modulus of nuclear matter and the stiffness of hyperon-nucleon interaction potential [62]. Shown in Fig. 9 is a comparison of the density dependence of pressure and the mass-radius relation of the neutron matter with the different slope parameter of symmetry energy. The constraint of symmetry energy from the pion production in heavy-ion collisions by the S π RIT data leads to the maximal mass of $2.0 M_\odot$ and radius of 11-13 km as shown the blue lines in Fig. 9 (b). The strange NS with the mass of $2.76 M_\odot$ and the radius of 15 km is obtained with the hard symmetry energy of $L=139$ MeV. The results are consistent with the NICER observations for PSR J0030+0451 and J0740+6620. It is obvious that the symmetry energies with the slope parameters of $L=35, 42, 53$ and 67 MeV result in the same maximal mass of NSs, but having the different radius. It should be noticed that we only include the neutrons, protons, electrons, muons and neutrinos as ingredient of NSs. The properties of symmetric nuclear matter and hyperon ingredients in the NSs influence the mass-radius relation.

IV. CONCLUSIONS

In summary, the isospin diffusion in the isotopic reactions of $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn} + ^{112}\text{Sn}$ at the incident energy of 270 MeV/nucleon is thoroughly investigated within the LQMD transport model. The symmetry energy manifests the opposite contribution on the N/Z and π^-/π^+ ratios in the low-density region and in the high-density domain. The hard symmetry energy enhances the N/Z ratio of free nucleons, but leads to the reduction of N/Z and π^-/π^+ ratios in the high-density region. The production of pions in heavy-ion collisions spreads the whole density range and most of pions are created in the dilute nuclear matter because of the rescattering processes of pions and resonances with nucleons. The stiffness of symmetry energy is sensitive to the π^- transverse spectra, but weakly impact the π^+ production. The pion optimal potential is obvious in the high-momentum regime and influences both the π^- and π^+ spectra. Systematics analysis with the symmetry energy and pion-nucleon potential at the density around $1.5\rho_0$ manifests the soft symmetry energy with the slope parameter $L(\rho_0) = 42 \pm 25$ MeV or stiffness parameter $\gamma_s = 0.3$ with the S π RIT data. More experiments are still ex-

pected for complementary constraining the high-density symmetry energy, i.e., the collective flows and differential flows of pions, the energy spectra of $^3\text{H}/^3\text{He}$ ratio etc. The neutron stars with the maximal mass of $2 M_\odot$ and radius of 11-13 km are obtained with the symmetry energy extracted from the pion data, which are consistent with the NICER observations for PSR J0030+0451 and J0740+6620.

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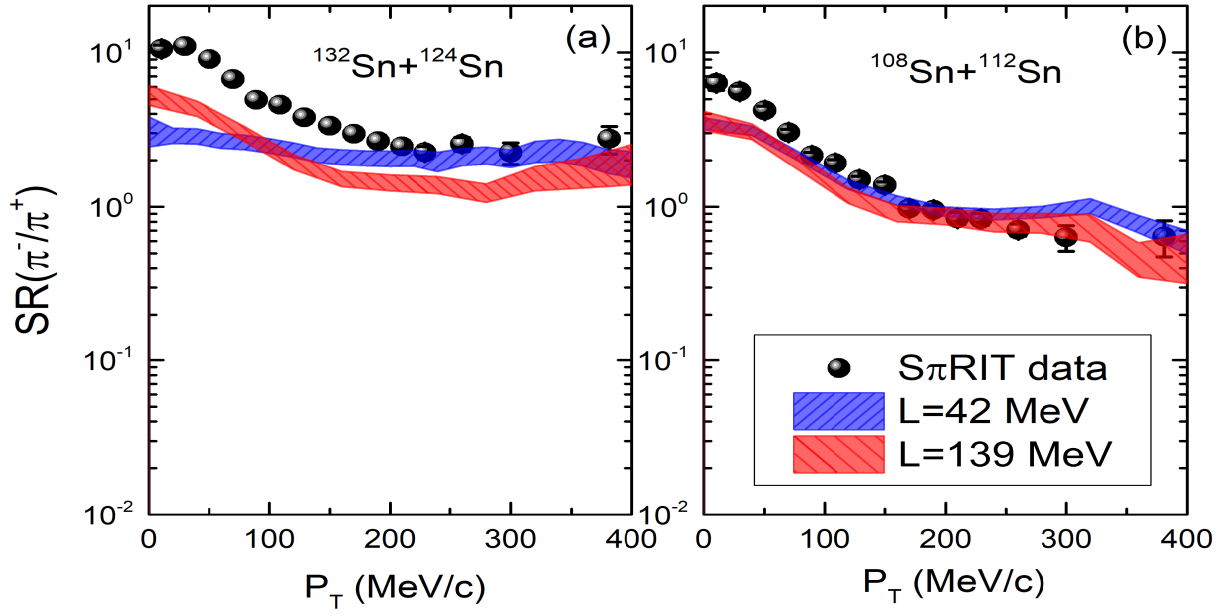


FIG. 7. The single ratio of π^-/π^+ in the isotropic reactions of (a) $^{132}\text{Sn} + ^{124}\text{Sn}$ and (b) $^{108}\text{Sn} + ^{112}\text{Sn}$ at 270A MeV and impact parameter of 3 fm with the different stiffness of symmetry energy.

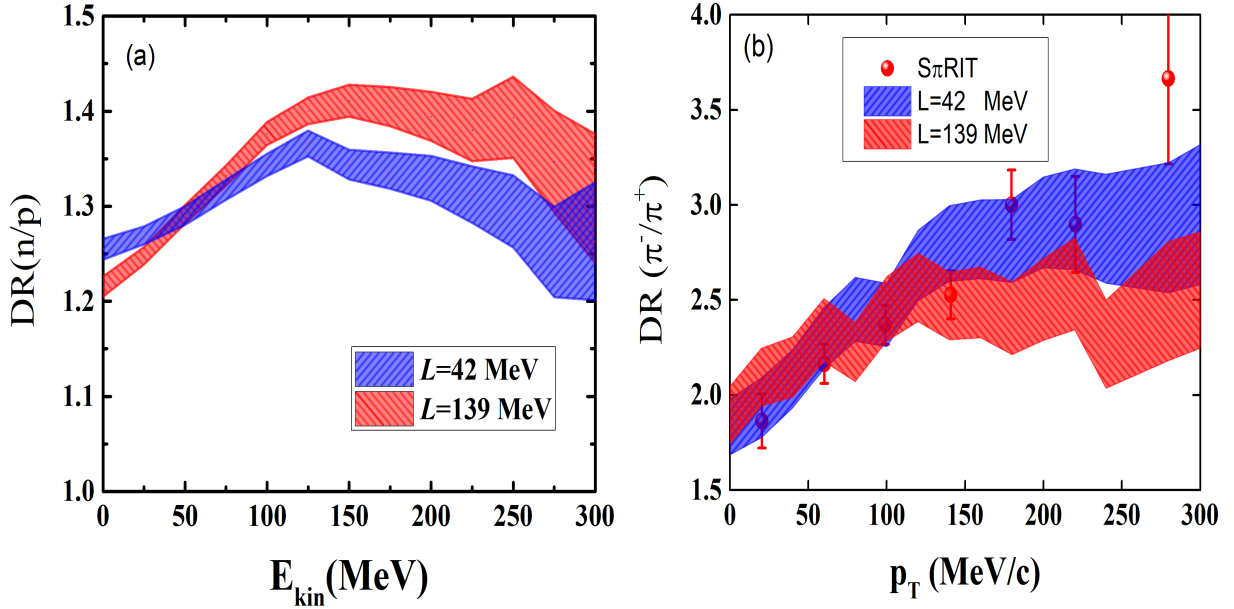


FIG. 8. The double ratios of (a) neutron/proton and (b) π^-/π^+ in the isotropic reactions of $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn} + ^{112}\text{Sn}$ at 270A MeV and impact parameter of 3 fm. The experimental data for the pion production are taken from the SπRIT collaboration [32].

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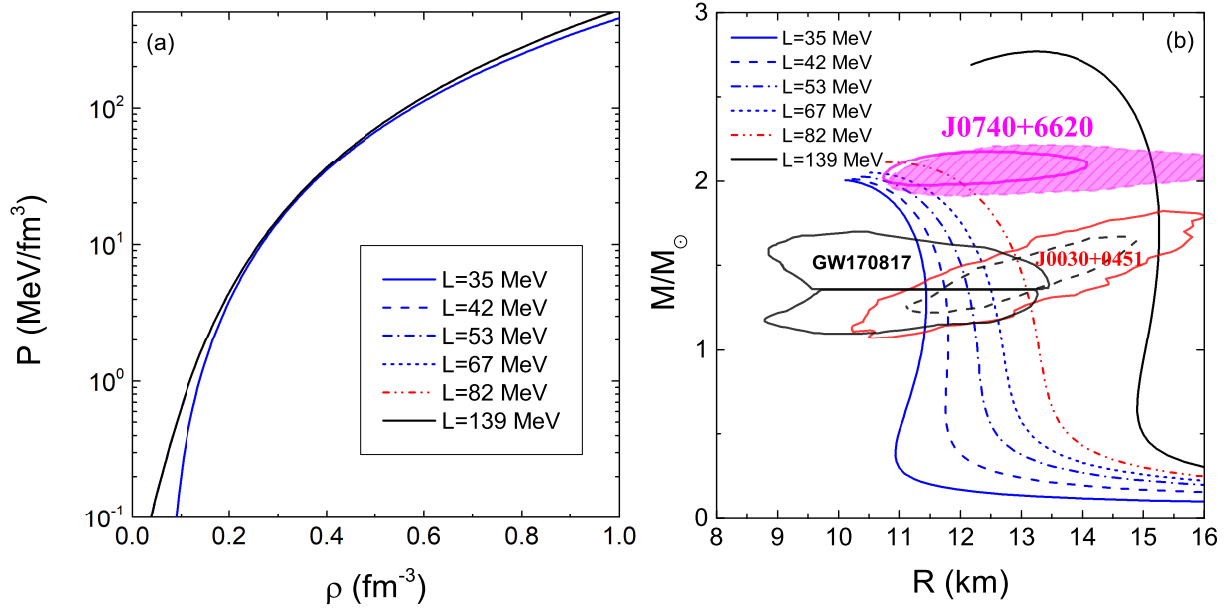


FIG. 9. (a) The pressure as a function of baryon density and (b) mass-radius relation of neutron star matter. The constraints from the GW170817 event [63, 64] and from the NICER observations for J0030+0451 [65, 66] and J0740+6620 [67, 68] are shown for comparison.

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